

**q^2 -Dependence of Meson Mixing in Few-Body Charge Symmetry Breaking:
 $\pi^o - \eta$ Mixing to One Loop in Chiral Perturbation Theory**

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ABSTRACT

It is pointed out that the meson mixing matrix elements usually considered responsible for the bulk of the observed few-body charge symmetry breaking are naturally q^2 -dependent in QCD. For $\pi^o - \eta$ mixing, using the usual representation of the pseudoscalar fields, the leading q^2 dependence can be explicitly calculated using chiral perturbation theory to one loop, the result being a significant decrease in the magnitude of the matrix element in going from timelike to spacelike values of q^2 . Since it is the latter range of q^2 which is relevant to NN scattering and the few-body bound state, this result calls into serious question the standard treatment of few-body charge symmetry breaking contributions associated with $\pi^o - \eta$ and $\rho - \omega$ mixing.

In the standard model, charge symmetry (CS) is broken not only by electromagnetism (EM) but also, through the inequality of up and down quark masses, by the strong interactions. This strong breaking leads to important non-Coulombic contributions to CS violating observables such as the difference of low energy nn and pp scattering amplitudes and the $A = 3$ binding energy difference, ΔBE .¹⁻³ It is generally believed that these non-Coulombic contributions can be accounted for in the meson exchange picture by isoscalar-isovector mixing (predominantly $\rho - \omega$, but also $\pi^0 - \eta$) in the intermediate meson propagators of one-boson exchange graphs.⁴⁻⁶ In the analysis of Refs 4-6, the $\rho - \omega$ matrix element is obtained from experimental data on $e^+e^- \rightarrow \pi^+\pi^-$ in the region of the $\rho - \omega$ interference shoulder, while the $\pi^0 - \eta$ matrix element is obtained by a pole model analysis of η, η' decays. The latter analysis (which has some additional shortcomings, to be discussed below) is feasible only because the mixing matrix elements are assumed to be q^2 -independent. This assumption is also implicit in the treatment of the $A = 3$ bound state and NN scattering, where time-like matrix elements, extracted as above, are used unchanged in the spacelike region. From the perspective of QCD, however, this assumption is clearly incorrect. Indeed, the basic hypothesis underlying the meson exchange picture is that, at low energies, QCD reduces to an effective hadronic theory involving mesons, nucleons, deltas (etc.). We know, however, that, as an effective low energy theory, such a hadronic theory must incorporate all possible terms, involving the low energy composite fields, which are not explicitly forbidden by the symmetries of the underlying theory (in this case, QCD). In particular, there will be terms higher order in derivatives, involving also the quark mass matrix, which will produce a q^2 -dependence to all isoscalar-isovector mixing matrix elements. How important such effects will be for a given CS-breaking (CSB) observable cannot be determined without either generating the effective theory from QCD or constraining the strengths of the relevant terms in the effective La-

grangian from experiment. Some information, however, already suggests that these effects are unlikely to be negligible. First, we know that, in the case of effective chiral Lagrangians, higher derivative terms typically occur scaled by a dimensionful factor $\Lambda_\chi \sim 4\pi f_\pi \sim 1\text{GeV}$.^{7–10} If this is true in general, one would expect significant changes in going from, eg., experimental data for $\rho - \omega$ at $q^2 \sim m_\rho^2$, to $q^2 < 0$. (We will consider the case of $\pi^0 - \eta$ mixing, using the chiral Lagrangian, explicitly below and see that this expectation is borne out.) Second, a recent model calculation¹¹, in which the q^2 -dependence of $\rho - \omega$ mixing arising from an intermediate (free) quark loop through the $u - d$ constituent quark mass difference is evaluated using a monopole *ansatz* for the meson-quark-antiquark vertex function, finds a very strong q^2 dependence of the $\rho - \omega$ element of the inverse vector meson propagator matrix. Such a q^2 dependence would translate into a significant q^2 dependence of the phenomenological $\rho - \omega$ mixing matrix element (this aspect of the calculation is not treated in Ref 11—see Ref 12 for a detailed discussion). This result is qualitatively compatible with the discussion above and, despite possible reservations about its potential model-dependence, makes clear the dangers inherent in the neglect of q^2 -dependence of the usual treatment.

One would, of course, ideally, like to reduce the model dependence of the $\rho - \omega$ result, since it is $\rho - \omega$ mixing which produces the dominant non-Coulombic CSB contribution to few-body observables (in the standard treatment). Unfortunately, one is hampered, in this regard, by a lack of information about higher order terms in the (putative) effective low-energy Lagrangian involving the vector meson fields. If we turn our attention to the next-to-leading contribution, associated with $\pi^0 - \eta$ mixing, however, we are in much better shape, since the effective low-energy Lagrangian relevant to the pseudoscalar sector is well-known, to order q^4 , as a result of the work of Gasser and Leutwyler^{8–10}. By computing $\pi^0 - \eta$ mixing to one-loop in this framework, one obtains the leading q^2 -dependence in a straightforward, and reliable,

fashion. Since the effective Lagrangian is not constrained beyond order q^4 , one cannot determine the higher order q^2 -dependent contributions, which restriction limits the validity of the results to the range $|q^2| < O(m_\eta^2)$, by the usual power counting arguments of the low-energy expansion.

Let us consider, therefore, $\pi^o - \eta$ mixing. Recall that the relevant terms in the effective chiral Lagrangian, to order q^4 , are given by⁸

$$\begin{aligned}
L_{eff} = & \frac{1}{4}f^2 Tr(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger) + \frac{1}{2}f^2 Tr[\mu M(\Sigma + \Sigma^\dagger)] + L_1 [Tr(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger)]^2 \\
& + L_2 Tr(\partial_\mu \Sigma \partial_\nu \Sigma^\dagger) Tr(\partial^\mu \Sigma \partial^\nu \Sigma^\dagger) + L_3 Tr(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma \partial_\nu \Sigma^\dagger \partial^\nu \Sigma) \\
& + L_4 Tr(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger) Tr[2\mu M(\Sigma + \Sigma^\dagger)] + L_5 Tr[2\mu(M\Sigma + \Sigma^\dagger M)\partial_\mu \Sigma^\dagger \partial^\mu \Sigma] \\
& + L_6 [Tr[2\mu M(\Sigma + \Sigma^\dagger)]]^2 + L_7 [Tr[2\mu M(\Sigma - \Sigma^\dagger)]]^2 \\
& + L_8 Tr[4\mu^2(M\Sigma M\Sigma + M\Sigma^\dagger M\Sigma^\dagger)]
\end{aligned} \tag{1}$$

where μ is a mass scale related to the value of the quark condensate⁸,

$$\Sigma = \exp(i\vec{\lambda} \cdot \vec{\pi}/f), \tag{2}$$

with $\vec{\lambda}$ the usual $SU(3)$ Gell-Mann matrices, $\vec{\pi}$ the pseudoscalar octet fields and f a dimensionful constant, equal to f_π in leading order. In Eq (1), M is the current quark mass matrix and we have set the external fields to zero. The (scale-dependent) coefficients L_1, \dots, L_8 are as defined in Ref 8. In what follows we ignore EM contributions to $\pi^o - \eta$ mixing, since these are known to vanish in the chiral limit¹³ and estimates for the effect of the departure from the chiral limit, based on the size of the relevant EM chiral logarithms¹⁴, show them to be negligible relative to the strong contributions resulting from L_{eff} in Eq (1). From Eq (1) one may compute the inverse $\pi_3 - \pi_8$ propagator matrix to one-loop as a function of q^2 (where q is the external four-momentum and π_3, π_8 are the unmixed octet fields that would be identical to the π^o, η fields, respectively, in the limit of exact isospin symmetry). Diagonalizing this matrix one obtains the q^2 -dependent mixing angle for the physical propagating modes

$$\begin{aligned} \theta(q^2) = & \frac{\sqrt{3}(m_d - m_u)}{4(m_s - \hat{m})} \left[1 - \frac{32\mu(m_s - \hat{m})}{f^2} (3L_7^r + L_8^r) + \frac{(-3\ell_\pi + \ell_\eta + 2\ell_K)}{32\pi^2 f^2} \right. \\ & \left. + \left(\frac{q^2 + m_\eta^2}{32\pi^2 f^2} \right) \left(1 + \frac{m_\pi^2}{(m_K^2 - m_\pi^2)} \ell n(m_\pi^2/m_K^2) \right) \right] \end{aligned} \quad (3)$$

where m_u, m_d, m_s are the u, d, s current quark masses, $\hat{m} = (m_u + m_d)/2$, L_i^r are the (scale-dependent) renormalized low-energy constants of Ref 8 and $\ell_P = m_P^2 \ell n(m_P^2/\mu_o^2)$, with P any of the pseudoscalar mesons and μ_o the renormalization scale. The q^2 -dependence of θ is unavoidable at the one-loop level. Following Ref 8 one may rewrite Eq (3) using one-loop results for other physical observables, obtaining

$$\begin{aligned} \theta(q^2) = & \frac{\sqrt{3}(m_{K^o}^2 - m_{K^+}^2)_{QCD}}{(m_K^2 - m_\pi^2)} \left[1 + \Delta_{GMO} \right. \\ & + \frac{1}{16\pi^2 f^2} \left(\frac{m_\eta^2}{(m_\eta^2 - m_\pi^2)} \right) (3m_\eta^2 \ell n(m_K^2/m_\eta^2) + m_\pi^2 \ell n(m_K^2/m_\pi^2)) \\ & \left. + \left(\frac{q^2 + m_\eta^2}{32\pi^2 f^2} \right) \left(1 + \frac{m_\pi^2}{(m_K^2 - m_\pi^2)} \ell n(m_\pi^2/m_K^2) \right) \right] \end{aligned} \quad (4)$$

where Δ_{GMO} is the (electromagnetically corrected) Gell-Mann-Okubo discrepancy

$$\Delta_{GMO} = (4m_K^2 - m_\pi^2 - 3m_\eta^2)/(m_\eta^2 - m_\pi^2). \quad (5)$$

In Eq (4), $(\Delta m_K^2)_{QCD}$ is the contribution to the $K^o - K^+$ mass-squared splitting resulting from $m_u \neq m_d$. This can be extracted from the physical splitting if one knows the corresponding EM contribution. The latter may be considered to be known if we accept Dashen's theorem¹³ for the EM self-energies,

$$(\Delta m_K^2)_{EM} = (\Delta m_\pi^2)_{EM} \quad (6)$$

since the quark-mass-difference contribution to the π splitting is $O[(m_d - m_u)^2]$ and known to be small¹⁵. Dashen's theorem is, however, valid only to lowest order in the EM chiral expansion. Constraints on the size of potential violations are discussed in

Ref 16. The resulting uncertainties in the size of $(\Delta m_K^2)_{EM}$ lead to a corresponding uncertainty of order 20% in the overall scale of the result (4). Apart from this overall scale uncertainty, the result (4) should provide an accurate representation of $\theta(q^2)$, subject to the restrictions of the validity of the low energy expansion, i.e. for values of $|q^2| < O(m_\eta^2)$.

Before proceeding, we must point out one important feature of the calculation which bears crucially on the proper interpretation of its results. Recall that L_{eff} is only one of a family of possible effective Lagrangians, all others being related to that in Eqn (1) by redefinitions of the octet fields which leave the one particle singularities, and hence the S-matrix elements, of the theory unchanged¹⁷⁾. The propagator matrix, and hence the mixing angle, $\theta(q^2)$, of course, depends on the particular definition of the octet fields employed: as such it is not a physical quantity. Only when combined with a full low-energy Lagrangian including the nucleon fields could one obtain results for physical processes (such as, eg., NN scattering) which were independent of the particular choice of the meson fields. In the context of few-body meson-exchange model calculations we are not, at present, able to carry out this program and, in fact, because of the insertion of phenomenological form factors for the meson-nucleon vertices, it is actually ambiguous as to exactly what definition of the meson fields is being used (in the sense above). The relevant question, in this setting, is then whether or not the q^2 dependence of the mixing (which must be present, on general grounds, for any choice of fields) is typically negligible or not. If we use the definition of the fields which leads to L_{eff} of the form given in Eqn (1), we find that q^2 dependence of the mixing is not negligible in few-body systems, and hence that the assumption of q^2 independence of the standard treatments is not a reasonable one to make. Unless, however, one formulates the entire scattering (or bound state) problem under consideration in a well-defined effective field theoretic manner, where

one can determine that the same choice for the meson fields is being used for the meson-nucleon vertices as in the purely mesonic sector, one is not guaranteed that simply plugging the result of Eqn (7) into an existing few-body code will lead to a reliable estimate of the CSB associated with $\pi^o - \eta$ mixing. This statement is, of course, equally true of the usual treatment, with, however, the added caveat that the usual treatment suffers from the absence of a demonstration that there even exists a choice of the meson fields for which the mixing is q^2 -independent. The result of Eqn (7) is, of course, perfectly reliable for the choice of meson fields implied by Eqn (1); the problem is that possible additional q^2 dependent CSB contributions associated with the meson-nucleon couplings cannot be *consistently* evaluated owing to the phenomenological treatment of the meson-nucleon vertices. For this reason, quantities such as the $\pi^o - \eta$ and $\rho - \omega$ matrix elements used in conventional few-body CSB calculations are actually ambiguous beyond leading order. This means that the appropriate (and most conservative) interpretation of the present calculation is that the difference between the results of using the q^2 -dependent mixing of Eqn (7) and the usual q^2 -independent mixing represents a lower bound for the *uncontrollable* theoretical uncertainties associated with the phenomenological modelling of the input meson-nucleon vertices. The comments which follow are to be viewed in this light.

From the results above we obtain, for the choice of fields implied by Eqn (1), two pieces of information: first, the size of the $\pi^o - \eta$ mixing and, second, its q^2 dependence.

Let us first consider the magnitude. The result (4) leads to a phenomenological mixing matrix element given by

$$m_{38}^2(q^2) = -(m_\eta^2 - m_\pi^2)\theta(q^2). \quad (7)$$

As mentioned above, there is an overall uncertainty in the result (7) as a consequence of the unknown size of possible violations of Dashen's theorem. Since this uncertainty

also enters the determination of Δ_{GMO} the effect is not quite linear in $(\Delta m_K^2)_{QCD}$, but it is very nearly so. EM and strong contributions to Δm_K^2 have the opposite sign so that EM contributions larger than those corresponding to Dashen's theorem lead to larger values of $(\Delta m_K^2)_{QCD}$. Let us write

$$m_{38}^2 = -[a_o + (q^2/m_\eta^2)a_1]. \quad (8)$$

The quantities a_o, a_1 are listed in Table 1 for a range of values of $(\Delta m_K)_{EM}$ between $1.3MeV$ (arising from Dashen's theorem) and $2.8MeV$ (which results from attempting to saturate the Cottingham formula for the kaon system with K and K^* intermediate states¹⁸).

The values of Table 1 should be considered to supercede those of earlier work on the subject for the following reasons. The $\pi^o - \eta$ matrix element of Ref 6c was extracted by an analysis of η, η' decays in which these decays are taken to be mediated by η, η' and π poles, intermediate $\pi^o\eta, \pi^o\eta'$ vertices being taken to be q^2 -independent. There are two problems with this analysis. First, as we have seen, the $\pi^o - \eta$ vertex (and hence, presumably, also the $\pi^o - \eta'$ vertex) is not q^2 -independent. Second, as one discovers from Ref 10, which treats $\eta \rightarrow 3\pi$ to one loop in chiral perturbation theory, a large portion of the 55% increase of the $\eta \rightarrow 3\pi$ amplitude in going from tree level to one-loop level is attributable to enhancements associated with final state interactions produced by the presence of s -wave $I = 0$ $\pi\pi$ pairs. As an s -wave $\pi\pi$ effect this enhancement can clearly not be represented by an intermediate pseudoscalar pole (the effect of the η' in L_{eff} is contained entirely in the low-energy constant L_7 ⁸⁾); it has, therefore, not been included in the pole model analysis. Its inclusion would presumably lower the extracted values of the $\pi^o - \eta$ and $\pi^o - \eta'$ matrix elements, even if one could treat them as q^2 -independent. There is, however, no need to perform a modified pole model analysis to extract the $\pi^o - \eta$ matrix element, since we can obtain

it directly from chiral perturbation theory, at least for the region of q^2 for which the low energy expansion is valid.*

Next we turn to the q^2 -dependence of m_{38}^2 . If we consider the change in the matrix element in going from $q^2 = m_\eta^2$ to $q^2 = -m_\eta^2$ (very nearly equal to the range over which the $\rho - \omega$ matrix element is extrapolated to get from the e^+e^- experimental data to the scattering region) we find that there is a decrease of 17%. One of the advantages of a model calculation would be that the model would provide also the higher order q^2 dependence; here we have access only to the leading (linear-in- q^2) dependence, though this dependence has (apart from the uncertainty in the overall scale, a feature subject to future improvement) the advantage of theoretical reliability, in the sense that the structure of the low-energy expansion of chiral perturbation theory is directly related to QCD and its convergence to one-loop has been tested for a large number of processes¹⁹. We see that the q^2 dependence of the mixing in the meson propagator is certainly not negligible. While one would like to quote numbers on how much this affects, eg., the uncertainty in the $\pi^0 - \eta$ contribution to the $A = 3$ binding energy difference (in the conservative sense described above), this is not possible because the binding energy difference is, in fact, sensitive to spacelike q^2 values considerably outside the range of validity of the low-energy expansion³. In order to make further

* It should be noted that the analysis of Refs 6a,6b which uses the quark model, together with certain results valid in the chiral limit, to evaluate, eg., the $\pi^0 - \eta$ matrix element, contains non-vanishing EM contributions which are incorrect. The EM contribution to $\pi^0 - \eta$ mixing actually vanishes in the chiral limit¹³. The error results from neglecting a class of graphs (“EM penguins of the second kind”¹⁹) which are required to properly implement all of the EM chiral constraints of Ref 13, in the chiral limit. The numerical consequence of this error are, however, not particularly significant.

progress one should, therefore, use the chiral results embodied in Eqs (7) and (8) to constrain model calculations. If an extension of the calculation of Ref 11 to the pseudoscalar sector were to reproduce the q^2 -dependence of the $\pi^o - \eta$ matrix element, this would considerably enhance our confidence in what it has to say about the running of the $\rho - \omega$ matrix element.

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Table 1. The constants a_o , a_1 as a function of $(\Delta m_K)_{EM}$ *

$(\Delta m_K)_{EM}$ (MeV)	a_o (GeV ²)	a_1 (GeV ²)
1.3	.00335	.00028
1.6	.00354	.00030
1.9	.00372	.00032
2.2	.00390	.00033
2.5	.00408	.00035
2.8	.00426	.00036

* All notation as in the text. a_o and a_1 are as defined in Eq (8).